

A Bessel Filter Crossover, and Its Relation to Others

- **Crossovers**
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- **Phase Shift**
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- **Bessels, 3dB Down**

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Introduction

One of the ways that a crossover may be constructed from a Bessel low-pass filter employs the standard low-pass to high-pass transformation. Various frequency normalizations can be chosen for best magnitude and polar response, although the linear phase approximation in the passband of the low-pass is not maintained at higher frequencies. The resulting crossover is compared to the Butterworth and Linkwitz-Riley types in terms of the magnitude, phase, and time domain responses.

A Brief Review of Crossovers

There are many choices for crossovers today, due especially to the flexibility of digital signal processing. We now have added incentive to examine unconventional crossover types. Each type has its own tradeoffs between constraints of flatness, cutoff slope, polar response, and phase response. See [1] and [2] for more complete coverage of crossover constraints and types. Much of the content of this paper is closely related to previous work by Lipshitz and Vanderkooy in [3].

Our sensitivity to frequency response flatness makes this one of the highest priorities. It is often used as a starting point when choosing a crossover type.

Cutoff slopes of at least 12 dB per octave are usually chosen because of limitations in the frequency range that drivers can faithfully reproduce. Even this is less than optimal for most drivers.

Polar response is the combined magnitude versus listening angle from noncoincident drivers [4]. The ideal case is a large lobe in the polar response directly in front of the drivers, and happens when low-pass and high-pass outputs are in-phase.

The phase response of a crossover is one of its most subtle aspects, and so is often ignored. A purely linear phase shift, which is equivalent to a time delay, is otherwise inaudible, as is a small non-linear phase shift. Still, there is evidence that phase coloration is audible in certain circumstances [5], and certainly some people are more sensitive to it than others.

A first-order crossover is unique, in that it sums with a flat magnitude response and zero resultant phase shift, although the low-pass and high-pass outputs are in phase quadrature (90°), and the drivers must perform over a huge frequency range. The phase quadrature that is characteristic of odd-order crossovers results in a moderate shift in the polar response lobe.

In spite of this, third-order Butterworth has been popular for its flat sound pressure *and* power responses, and 18 dB per octave cutoff slope.

Second-order crossovers have historically been chosen for their simplicity, and a usable 12 dB per octave cutoff.

Fourth-order Linkwitz-Riley presents an attractive option, with flat summed response, 24 dB per octave cutoff, and outputs which are always in phase with each other, producing optimal polar response.

Steeper cutoff slopes are known to require higher orders with greater phase shift, which for the linear phase case is equivalent to more time delay.

A number of other novel and useful designs exist which should be considered when choosing a crossover. Generating the high-pass output by subtracting the low-pass output from an appropriately time-delayed version of the input results in a linear phase crossover, with tradeoffs in cutoff slope, polar response, and flatness [1]. Overlapping the design frequencies and equalizing the response can result in a linear phase crossover [3], with a tradeoff in polar response. A crossover with perfect polar response can be designed with a compromise in phase response or cutoff slope [6].

What is a Bessel Crossover?

The Bessel filter was not originally designed for use in a crossover, and requires minor modification to make it work properly. The purpose of the Bessel filter is to achieve approximately linear phase, linear phase being equivalent to a time delay. This is the best phase response from an audible standpoint, assuming you don't want to correct an existing phase shift.

Bessels are historically low-pass or all-pass. A crossover however requires a separate high-pass, and this needs to be derived from the low-pass. There are different ways to derive a high-pass from a low-pass, but here we discuss a natural and traditional one that maximizes the cutoff slope in the high-pass. Deriving this high-pass Bessel, we find that it no longer has linear phase. Other derivations of the high-pass can improve the combined phase response, but with tradeoffs.

Two other issues closely related to each other are the attenuation at the design frequency and the summed response. The traditional Bessel design is not ideal here. We can easily change this by shifting the low or high-pass up or down in frequency. This way, we can adjust the low-pass vs. high-pass response overlap, and at the same time achieve a phase difference between the low-pass and high-pass that is nearly constant over all frequencies. In the fourth order case this is 360°, or essentially in-phase. In fact, the second and fourth order cases are comparable to a Linkwitz-Riley with slightly more rounded cutoff!

Bessel Low-Pass and High-Pass Filters

The focus of this paper is on crossovers derived using traditional methods, which begin with an all-pole low-pass filter with transfer function (Laplace Transform) of the form $1/p(s)$, where $p(s)$ is a polynomial whose roots are the poles.

The Bessel filter uses a $p(s)$ which is a Bessel polynomial, but the filter is more properly called a Thomson filter, after one of its developers [7]. Still less known is the fact that it was actually reported several years earlier by Kiyasu [8].

Bessel low-pass filters have maximally flat group delay about 0 Hz [9], so the phase response is approximately linear in the passband, while at higher frequencies the linearity degrades, and the group delay drops to zero (see Fig. 1 and 2). This nonlinearity has minimal impact because it occurs primarily when the output level is low. In fact, the phase response is so close to a time delay that Bessel low-pass and all-pass filters may be used solely to produce a time delay, as described in [10].

The high-pass output transfer function may be generated in different ways, one of which is to replace every instance of s in the low-pass with $1/s$. This “flips” the magnitude response about the design frequency to yield the high-pass. Characteristics of the low-pass with respect to 0 Hz are, in the high-pass, with respect to infinite frequency instead. A number of other high-pass derivations are possible, but they result in compromised cutoff slope or polar response (see [1]). These are beyond the scope of this paper.

This popular method results in the general transfer function (1); (2) is a fourth-order Bessel example.

$$\frac{1}{c_0 + c_1 \left(\frac{1}{s}\right) + c_2 \left(\frac{1}{s}\right)^2 + \dots + c_n \left(\frac{1}{s}\right)^n} = \frac{s^n}{c_n + c_{n-1} \cdot s + c_{n-2} \cdot s^2 + \dots + c_0 \cdot s^n} \quad (1)$$

$$\frac{1}{1 + \left(\frac{1}{s}\right) + \frac{9}{21} \left(\frac{1}{s}\right)^2 + \frac{2}{21} \left(\frac{1}{s}\right)^3 + \frac{1}{105} \left(\frac{1}{s}\right)^4} = \frac{s^4}{105 + \frac{2}{21} \cdot s + \frac{9}{21} \cdot s^2 + s^3 + s^4} \quad (2)$$

Note the reversed coefficient order of the high-pass as compared to the low-pass, once it's converted to a polynomial in s , and an added n^{th} -order zero at the origin. This zero has a counterpart in the low-pass, an implicit n^{th} -order zero at infinity! The nature of the response of the high-pass follows from equation (3) below, where s is evaluated on the imaginary axis to yield the frequency response.

$$s = j\omega, \quad p\left(\frac{1}{j\omega}\right) = p(-j\omega_h), \quad \omega_h = \frac{1}{\omega} \quad (3)$$

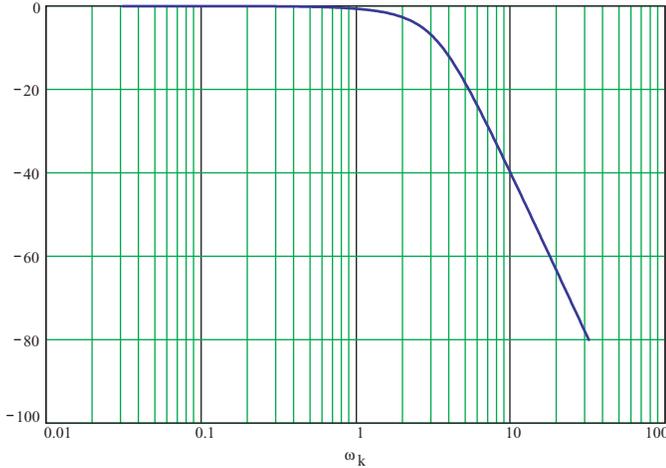


Figure 1. Fourth-Order Bessel Magnitude

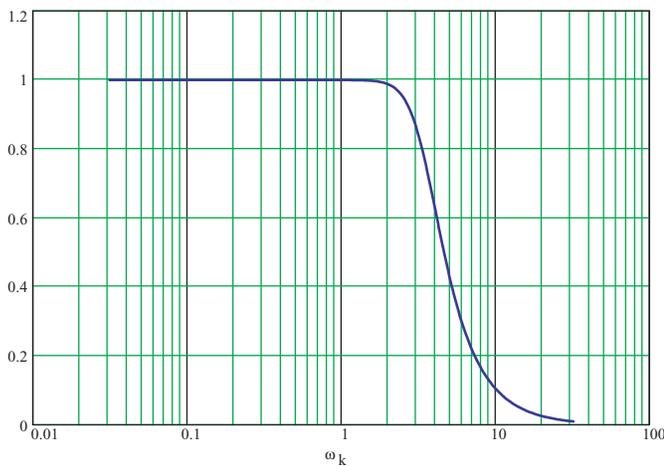


Figure 2. Fourth-Order Bessel Group Delay

The magnitude responses of the low-pass and the high-pass are mirror images of each other on a log-frequency scale; the negative sign has no effect on this. The phase of the low-pass typically *drops* near the cutoff frequency from an asymptote of zero as the frequency is *increased*, and asymptotically approaches a negative value. However, in addition to being mirror images on a log-frequency scale, the phase of the high-pass is the negative of the low-pass, which follows from the negative sign in (3). So the phase *rises* from zero at high frequency, and approaches a positive value asymptotically as the frequency is *decreased*. This results in offset curves with similar shape. Any asymmetry of the s-shaped phase curve is mirrored between the low-pass and high-pass. See Figure 5 for a second-order example, where the phase curve also has inherent symmetry.

One special case is where the denominator polynomial $p(s)$ has symmetric coefficients, where the n^{th} coefficient is equal to the constant term; the $(n-1)^{\text{st}}$ coefficient is equal to the linear term, etc. This is the case for Butterworth and therefore the Linkwitz-Riley types [3]. A fourth-order Linkwitz-Riley is given as an example in equation (4).

$$\frac{1}{1 + 2\sqrt{2}\cdot s + 4\cdot s^2 + 2\sqrt{2}\cdot s^3 + s^4} \quad (4)$$

When this is the case, coefficient reversal has no effect on $p(s)$, and the high-pass differs from the low-pass only in the numerator term s^n . This numerator can easily be shown to produce a constant phase shift of 90° , 180° , 270° , or 360° (360° is in-phase in the frequency domain), with respect to the low-pass, when frequency response is evaluated on the imaginary axis. For the second-order case $s^2=(j\omega)^2 = -\omega^2$ and the minus sign indicates a polarity reversal (or 180° phase shift at all frequencies).

Normalizations

Filter transfer functions are normalized by convention for $\omega_0=1, (f = \frac{1}{2\cdot\pi} \text{ Hz})$

and are then designed for a particular frequency by replacing every instance of s in the transfer function by $\frac{s}{\omega_0}, \omega_0 = 2\cdot\pi\cdot f_0$

This has the effect of shifting the magnitude and phase responses right or left when viewed on a log-frequency scale. Of course, it doesn't affect the shapes of these response curves, since when the transfer functions are evaluated:

$$f\left(\frac{s}{\omega_0}\right) = f\left(\frac{j\omega}{\omega_0}\right) = f(jy), \quad y = \frac{\omega}{\omega_0} \quad (5)$$

where y is a constant multiple of the variable frequency. The group delay, being the negative derivative of the phase with respect to angular frequency, is also scaled up or down.

This process can also be used to adjust the overlap between the low-pass and high-pass filters, so as to modify the summed response. After this is done, the filters are still normalized as before, and may be designed for a particular frequency. Adjusting the overlap will be done here with a normalization constant u , which will be applied equally but oppositely to both the low-pass and high-pass. In the low-pass, s is replaced by (s/u) , and in the high-pass, s is replaced by (su) . The low-pass response is shifted right ($u > 1$) or left ($u < 1$) when viewed on a log frequency scale, and the high-pass response is shifted in the opposite direction.

These overlap normalizations may be based on the magnitude response of either output at the design frequency, chosen for the flattest summed response, for a particular phase shift, or any other criterion.

Normalization influences the symmetry of $p(s)$, but perfect symmetry is not achievable in general. This means that it will not always be possible to make the low-pass and high-pass phase response differ *exactly* by a constant multiple of 90° for some normalization. The situation can be clarified by normalization for $c_n = 1$, as done by Lipshitz and Vanderkooy in [1] and [5], where $c_0 = 1$ for unity gain at 0 Hz. This form reveals any inherent asymmetry. Equation (6) shows the general low-pass, while (7) is the fourth-order Bessel denominator. Note that it becomes *nearly* symmetric, and relatively similar to the Linkwitz-Riley in (4).

$$\frac{1}{c_0 + c_1\cdot s + c_2\cdot s^2 + \dots + c_n\cdot s^n} \xrightarrow{s \rightarrow \frac{s}{\sqrt[n]{c_n}}} \frac{1}{1 + k_1\cdot s + k_2\cdot s^2 + \dots + k_n\cdot s^n} \quad (6)$$

$$1 + s + \frac{9}{21}\cdot s^2 + \frac{2}{21}\cdot s^3 + \frac{1}{105}\cdot s^4 \longrightarrow 1 + 3.2011\cdot s + 4.3916\cdot s^2 + 3.1239\cdot s^3 + s^4 \quad (7)$$

Phase-Matched Bessels

The textbook low-pass Bessel is often designed for an approximate time delay of $t_o = \frac{1}{\omega_o}$

rather than for the common -3 dB or -6 dB level at the design frequency used for crossovers. This design is used as a reference to which other normalizations are compared. The low-pass and high-pass have quite a lot of overlap, with very little attenuation at the design frequency, as shown in Figure 3, for a second-order Bessel with one output inverted.

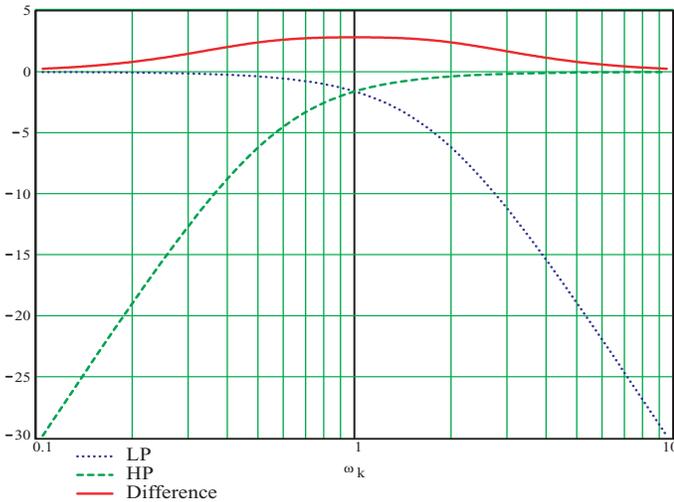


Figure 3. Second-Order Bessel Crossover

Bessel polynomials of degree three or higher are not inherently symmetric, but may be normalized to be nearly symmetric by requiring a phase shift at the design frequency of 45° per order, negative for the low-pass, positive for the high-pass. This results in a fairly constant relative phase between the low-pass and high-pass at all other frequencies. Equation (8) shows an equation for deriving the normalization constant of the fourth-order Bessel, where the imaginary part of the denominator (7) is set to zero for 180° phase shift at the design frequency.

$$s = j\omega_p, \quad \omega_p - \frac{2}{21} \cdot \omega_p^3 = 0, \quad u = \frac{1}{\omega_p} = \frac{1}{\sqrt{10.5}} \quad (8)$$

This normalization is not new, but was presented in a slightly different context in [5], with a normalization constant of 0.9759, which is the square of the ratio of the phase-match u in equation (8) to the u implied by equations (6) and (7), the fourth root of $1/105$.

Since the phase nonlinearity of the high-pass is now in the passband, the crossover resulting from the sum of the two approaches phase linearity only at lower frequencies. This doesn't preclude it from being a useful crossover.

The summed magnitude response of the Bessel normalized by the 45° is fairly flat, within 2 dB for the second-order and fourth-order. We may adjust the overlap slightly for flattest magnitude response instead, at the expense of the polar response. Figures 4-6 show the results of four normalizations for the second-order filter. The -3 dB and phase-match normalizations are illustrated in Figures 5 and 6. Note that for the second-order phase-match design, low-pass and high-pass group delays are exactly the same.

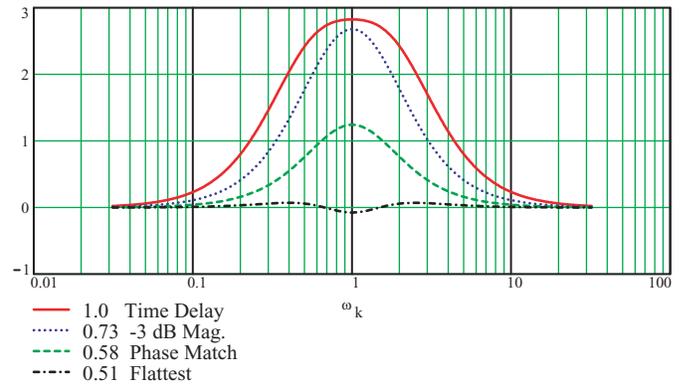


Figure 4. Comparison of Second-Order Bessel Sums

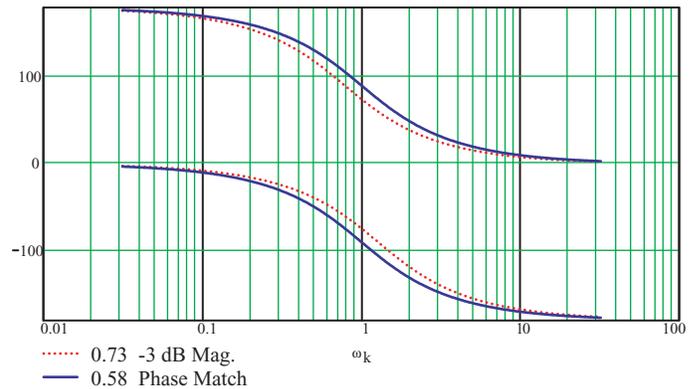


Figure 5. Comparison of Second-Order Phase

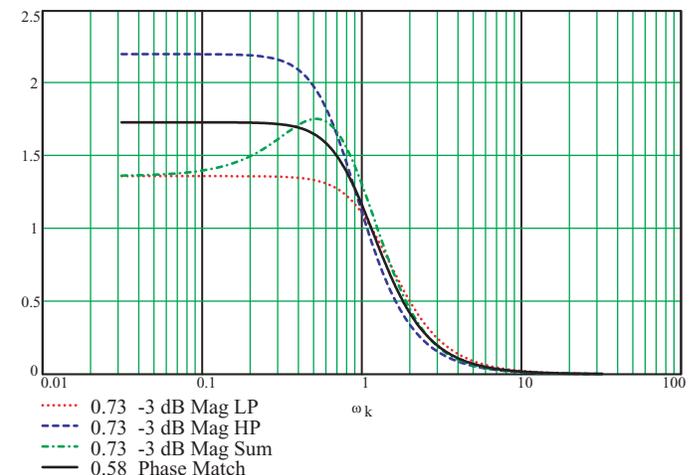


Figure 6. Second-Order Group Delay

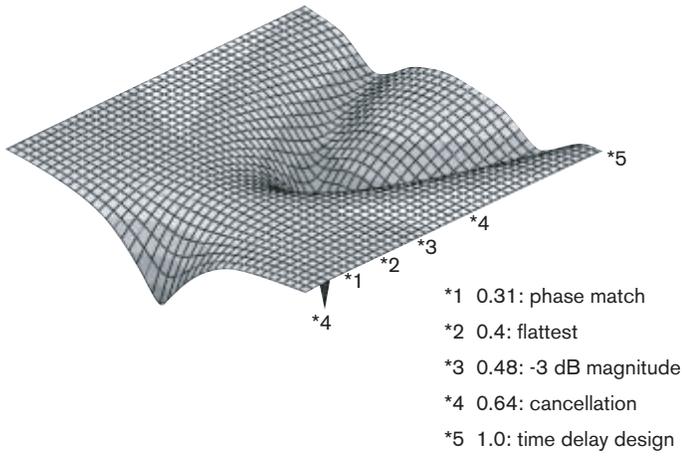


Figure 7. Summed Fourth-Order Bessel Frequency Response vs. Normalization. Normalization values are relative to time delay design.

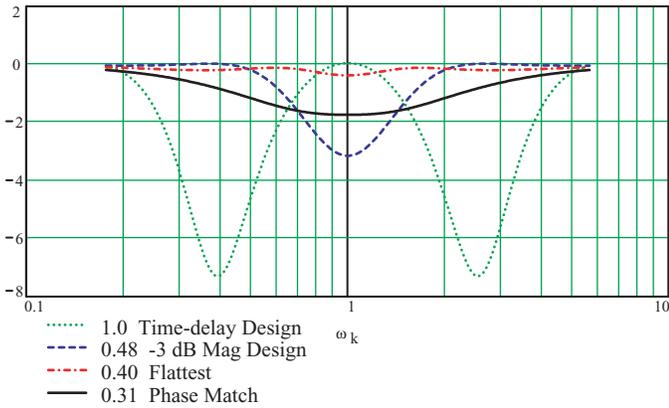


Figure 8. Summed Fourth-Order Responses

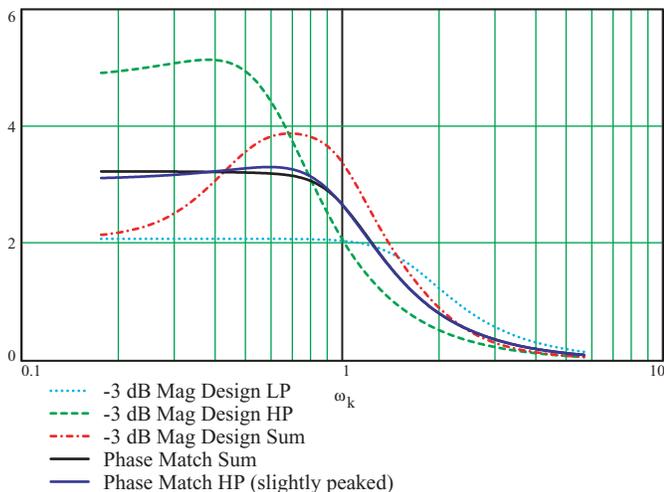


Figure 9. Fourth-Order Sum Group Delays

The fourth-order is illustrated in Figures 7-9, Figure 7 being a 3-D plot of frequency response versus normalization. Figure 8 shows four cases, which are cross-sections of Figure 7. The phase-match case has good flatness as well as the best polar response. The fourth-order Linkwitz-Riley is very similar to the Bessel normalized by 0.31. The third-order Bessel magnitude has comparable behavior.

In a real application, phase shifts and amplitude variations in the drivers will require some adjustment of the overlap for best performance. The sensitivity of the crossover response to normalization should be considered [2].

Comparison of Types

Butterworth, Linkwitz-Riley, and Bessel crossovers may be thought of as very separate types, while in fact they are all particular cases in a continuous space of possible crossovers. The separate and summed magnitude responses are distinct but comparable, as can be seen by graphing them together (Figure 10). The Bessel and Linkwitz-Riley are the most similar. The Butterworth has the sharpest initial cutoff, and a +3dB sum at crossover. The Linkwitz-Riley has moderate rolloff and a flat sum. The Bessel has the widest, most gradual crossover region, and a gentle dip in the summed response. All responses converge at frequencies far from the design frequency.

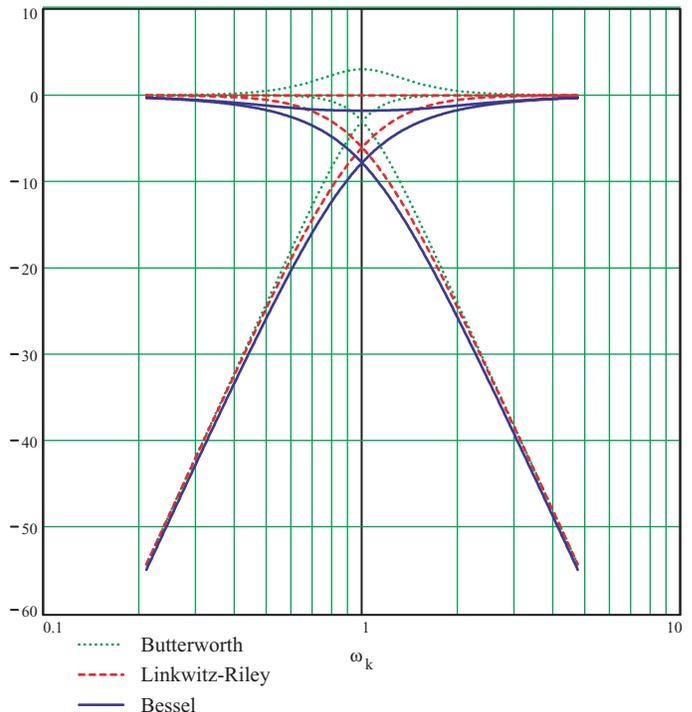


Figure 10. Fourth-Order Magnitudes

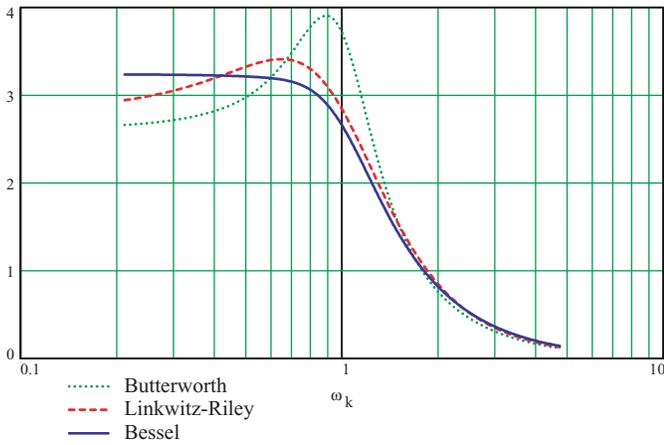


Figure 11. Fourth-Order Group Delays

The phase responses also look similar, but the amount of peaking in the group delay curve varies somewhat, as shown in Figure 11. There is no peaking in the Bessel low-pass, while there is a little in the high-pass for orders > 2. The summed response has only a little peaking. The group delay curve is directly related to the behaviour in the time domain, as discussed in [11]. The most overshoot and ringing is exhibited by the Butterworth design, and the least by the Bessel.

Often when discussing crossovers, the low-pass step response is considered by itself, while the high-pass and summed step response is usually far from ideal, except in the case of the linear phase crossover; this has been known for some time [12], but step-response graphs of higher-order crossovers are generally avoided out of good taste!

Table 1 gives Bessel crossover denominators normalized for time delay and phase match. Note the near-perfect symmetry for the (last three) phase-match cases.

Drag Net Bessel Crossover at -3 dB

Rane's Bessel crossover is set for phase match between low-pass and high-pass. This minimizes lobing due to driver separation, and also results in a pretty flat combined response. Another popular option is to have the magnitude response -3 dB at the design frequency. If -3 dB is desired at the setting, the frequency settings need to be changed by particular factors.

You will need to enter separate values for low-pass and high-pass: multiply the low-pass and high-pass frequencies by the following factors:

Crossover Type	Low-Pass	High-Pass
Second-Order (12 dB/octave)	1.272	0.786
Third Order (18 dB/octave)	1.413	0.708
Fourth Order (24 dB/octave)	1.533	0.652

For example, for a second-order low-pass and high-pass set to 1000 Hz, set the low-pass to 1272 Hz and the high-pass to 786 Hz.

Normalization with respect to time delay design	Low-pass Denominator Polynomial	High-pass Denominator Polynomial
1	$1 + s + \frac{1}{3} \cdot s^2$	$\frac{1}{3} + s + s^2$
1	$1 + s + \frac{2}{5} \cdot s^2 + \frac{1}{15} \cdot s^3$	$\frac{1}{15} + \frac{2}{5} \cdot s + s^2 + s^3$
1	$1 + s + \frac{9}{21} \cdot s^2 + \frac{2}{21} \cdot s^3 + \frac{1}{105} \cdot s^4$	$\frac{1}{105} + \frac{2}{21} \cdot s + \frac{9}{21} \cdot s^2 + s^3 + s^4$
$\frac{1}{\sqrt{3}} = 0.5774$	$1 + \sqrt{3} \cdot s + s^2$	$1 + \sqrt{3} \cdot s + s^2$
0.4030	$1 + 2.481 \cdot s + 2.463 \cdot s^2 + 1.018 \cdot s^3$	$1.018 + 2.463 \cdot s + 2.481 \cdot s^2 + s^3$
$\frac{1}{\sqrt{10.5}} = 0.3086$	$1 + 3.240 \cdot s + 4.5 \cdot s^2 + 3.240 \cdot s^3 + 1.050 \cdot s^4$	$1.050 + 3.240 \cdot s + 4.5 \cdot s^2 + 3.240 \cdot s^3 + s^4$

Table 1 - Bessel Crossovers of Second, Third, and Fourth-Order, Normalized First for Time Delay Design, then for Phase Match at Crossover

Summary

A Bessel crossover designed as described above is not radically different from other common types, particularly compared to the Linkwitz-Riley. It does not maintain linear phase response at higher frequencies, but has the most linear phase of the three discussed, along with fairly good magnitude flatness and minimal lobing for the even orders. It is one good choice when the drivers used have a wide enough range to support the wider crossover region, and when good transient behaviour is desired.

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