

Bandwidth in Octaves Versus Q in Bandpass Filters

- Given -3 dB Points to Find BW and Q
- Given BW in Octaves to Find Q
- Given Q to Find BW in Octaves

Dennis Bohn
Rane Corporation

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A generalized treatment is presented for the mathematical relationships that exist between Q and bandwidth expressed in octaves for bandpass filters. Closed solutions for each relationship are given along with convenient tables. A Windows® Excel® program for calculating BW (octaves) and Q is available for downloading at www.rane.com/note170.html.

Introduction

Analog and digital audio designers confront daily every imaginable aspect of active and passive filters. Most often, these are bandpass filters partially characterized by a quality factor, Q, and a bandwidth, BW. There exists enough books on active and passive filter design to fill a modest office library, however, there are certain aspects of the relationship between bandwidth *as expressed in octaves* versus Q that are poorly documented — if at all.

The purpose of this article is to derive the mathematical relationships between these two variables and provide a useful “look-up” table for each, as well as a handy Excel™ spreadsheet program, which may be downloaded from www.rane.com/note170.html.

¹ This is an expanded version of an article titled “Bandpass Filter Design” originally published in Studio Sound magazine January, 1983, and reprinted in S&VC magazine, August, 1999.

Definitions and Review

A bandpass filter is characterized by three major parameters: center frequency, amplitude response (gain), and bandwidth. *Center frequency* is the frequency at which the amplitude is maximum; *gain* is the maximum amplitude response occurring at the center frequency; and *bandwidth* (or *passband*) is the frequency range between the -3 dB points located on either side of the center frequency. Bandwidth is expressed in several ways: in *frequency*, as being so many Hertz wide; or in *octaves*, as being so many octaves (or fractional octave) wide; or in *decades*, as being so many decades (or fractional decade) wide. Far and away, the most common audio usage is to express bandwidth in octaves. It is here that the literature falls short in giving sufficient mathematical relationships to allow answers to be expressed easily in “octaves.”

When first designing an audio filter, normally the required BW in octaves is known and the associated Q needs to be calculated; once the filter has been designed than Q is easily calculated by measuring the -3 dB frequency points, taking the difference, and dividing that into the center frequency; and lastly, the BW in octaves is then calculated.

A third situation arises where *only* Q is known and the BW in octaves is desired. *This calculation is not obvious — nor easy.* The next section presents the necessary closed solutions for each of these calculations. For reference purposes, Figure 1 shows a bandpass filter with its associated parameters labeled for clarity and is used for derivation purposes.

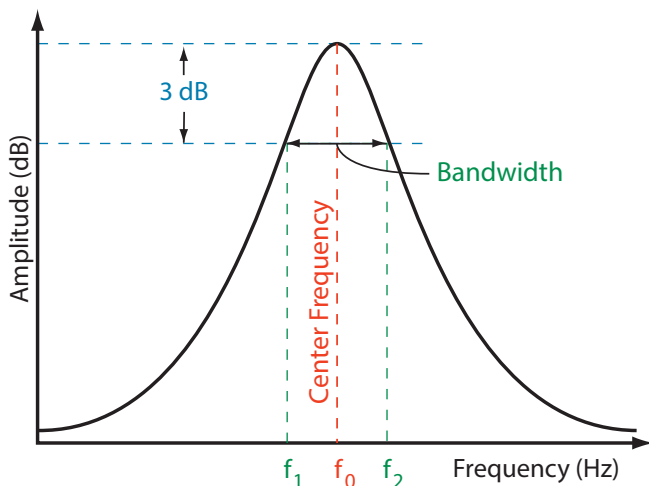


Figure 1. Bandpass Filter Parameters

Given the -3 dB Points, To Find BW and Q

If the -3 dB points are known, then calculating the BW in octaves is straightforward:

Let

$$f_2 = y f_1,$$

where y is any positive real number.

Define N as the number of octaves of BW, i.e.,

N octaves means that

$$N = \frac{\log y}{\log 2},$$

then

$$y = 2^N - 1 \quad (1)$$

Solving for N gives

$$N = \frac{\log y}{\log 2} \quad (2)$$

and, by definition

$$f_0 = \sqrt{f_1 f_2} \quad (3)$$

Given BW in Octaves, To Find Q

If the BW in octaves is known without knowledge of the actual -3 dB frequencies, and Q is to be calculated, then the following development leads to the required formula:

In general, f_0 is the geometric mean of the skirt frequencies, f_1 and f_2 :

therefore

$$f_0 = \sqrt{f_1 f_2}.$$

From (1)

$$y = 2^N f_1 - f_1$$

or

$$y = Q^2 (y^2 - 2y + 1).$$

From (3) and (1)

$$Q = \frac{\sqrt{2^N} f_1}{2^N f_1 - f_1}$$

or

$$Q = \frac{\sqrt{2^N}}{2^N - 1} \quad (4)$$

Table 1 shows several examples of equation (4) for BW's commonly used in audio design work.

BW (octaves)	Q
2.0	0.667
1.0	1.414
2/3	2.145
1/2	2.871
1/3	4.318
1/6	8.651
1/10	14.424
1/30	43.280

Table 1. Q Versus BW (octaves)

Given Q, To Find BW in Octaves

If only Q is known and the BW in octaves is desired then equation (4) must be turned around and re-expressed in terms of Q:

From (4)

$$Q = \frac{\sqrt{2^N}}{2^{\frac{N}{2}} - 1}$$

or

$$\sqrt{2^N} = Q(2^{\frac{N}{2}} - 1)$$

Since

$$y = 2^{\frac{N}{2}}$$

then

$$\sqrt{y} = Q(y - 1)$$

squaring gives

$$y = Q^2(y^2 - 2y + 1)$$

rearranging

$$y^2 - \left(\frac{2Q^2 + 1}{Q^2}\right)y + 1 = 0$$

applying quadratic solution:

$$y = \frac{2Q^2 + 1}{2Q^2} \pm \sqrt{\left(\frac{2Q^2 + 1}{Q^2}\right)^2 - 4} - 1 \quad (5)$$

N (the number of octaves of BW) is now found from equation (2). [See note 3 for a simpler form.]

Table 2 gives a handy look-up reference of BW (octaves) versus Q.

Q	BW (oct)	Q	BW (oct)	Q	BW (oct)
0.50	2.54	1.50	.945	6.50	.222
0.55	2.35	1.60	.888	7.00	.206
0.60	2.19	1.70	.837	7.50	.192
0.65	2.04	1.80	.792	8.00	.180
0.667	2.00	1.90	.751	8.50	.170
0.70	1.92	2.00	.714	8.65	.167
0.75	1.80	2.15	.667	9.00	.160
0.80	1.70	2.50	.573	9.50	.152
0.85	1.61	2.87	.500	10.0	.144
0.90	1.53	3.00	.479	15.0	.096
0.95	1.46	3.50	.411	20.0	.072
1.00	1.39	4.00	.360	25.0	.058
1.10	1.27	4.32	.333	30.0	.048
1.20	1.17	4.50	.320	35.0	.041
1.30	1.08	5.00	.288	40.0	.036
1.40	1.01	5.50	.262	45.0	.032
1.414	1.00	6.00	.240	50.0	.029

Table 2. BW (octaves) Versus Q

Bandwidth vs. Q Calculator

A Microsoft Excel® worksheet is available for downloading from rane.com. It's at the bottom of the page for this RaneNote: rane.com/note170.html.

²The squaring operation introduces an extraneous root appearing as the "minus" square root term. The "plus" square root term gives the correct answer. The "minus" square root term gives the reciprocal answer, i.e., the "plus" answer leads to y, while the "minus" answer yields 1/y.

³A simpler formula for determining the BW in octaves, given Q, appears on page 5 of RaneNote 167 *Why DSP Boxes Set the Same Way Differ* by Ray Miller. The simpler formula does not introduce the +/- ambiguity. [Thanks to Robert Bristow-Johnson for pointing this out.]